Presumably, this relationship is independent of the possible nonequilibrium effects. A simultaneous pitot-pressure (p_0') measurement was obtained with each drag measurement; thus the presented drag coefficients are proportional to a ratio of measured quantities.

It is noted from Fig. 1, where comparisons are made with the bounds of previously published data,[1] that the present data are appreciably below the earlier low-density data. This drag reduction is believed due to the relatively colder wall situation for the present data (i.e., the wall-to-total stagnation temperature ratio is less than $\frac{1}{2}$ the value for the earlier data). The present data do not reveal a significant influence of geometry in this low-density regime. This result was also previously noted from the earlier data[1] published by the authors.

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Exact Expressions for Composition of Gas Mixture at Equilibrium

C. P. Kentzer* and W. A. Anderson†

Douglas Aircraft Company, Inc., Santa Monica, Calif.

TEMPERATURE T and density ρ are usually chosen as the two independent variables describing the state of a gas at equilibrium. In practical problems, only the total enthalpy is known, whereas the second independent variable and the composition of the mixture must be determined as a part of the solution. For this reason, the enthalpy and temperature are the most convenient variables for use in homenthalpic flow calculations.

A study of an iterative procedure for the determination of equilibrium composition led to the discovery that closed-form, exact solutions are possible in some simple cases. As an example, consider a five-component mixture of O, N, O_2, N_2, and NO participating in the following three elementary reactions:

$$O_2 \rightleftharpoons 2O \qquad N_2 \rightleftharpoons 2N \qquad NO \rightleftharpoons O + N \tag{1}$$

If n_i are the specific molar concentrations (moles of i per gram of mixture), then the atom conservation equations may be written as

$$n_O + 2n_{O_2} + n_{NO} = C_1 \tag{2}$$

$$n_N + 2n_{N_2} + n_{NO} = C_2 \tag{3}$$

where C_1 and C_2 are constants. The law of mass action gives

$$n_O^2/n_{O_2} = (1/\rho)K_1 \tag{4}$$

$$n_N^2/n_{N_2} = (1/\rho)K_2 \tag{5}$$

$$n_O n_N/n_{NO} = (1/\rho)K_3 \tag{6}$$

where ρ is the density, and K_1, K_2, and K_3 are equilibrium concentration constants, known functions of temperature only. The enthalpy may be written as

$$h = v_O n_O + v_N n_N + v_{O_2} n_{O_2} + v_{N_2} n_{N_2} + v_{NO} n_{NO} \tag{7}$$

where

$$v_i = (\partial h/\partial n_i)_{T,\, n_j}$$

and where the partial derivative is taken keeping temperature T and concentrations n_j, $j \neq i$, constant. The functions v_i are known functions of temperature only. Considering h and T as given, we have six equations, Eqs. (2–7), for the determination of six unknowns, namely, the density ρ and the five concentrations of n_i. Equations (2, 3, and 7) are linear in the n_i's. If the nonlinear law-of-mass-action equations are not too complicated, one may proceed as follows. Substituting density from Eq. (6) into Eqs. (4) and (5) and introducing as a parameter a new unknown

$$x = n_N/n_O \tag{8}$$

one can use Eqs. (8 and 2–5) to express the unknown concentrations in terms of x. The result is

$$\left.\begin{aligned} n_O &= \frac{2C_1K_1K_3x^2 + K_1K_2(C_1 - C_2)x - 2C_2K_2K_3}{[(2K_3 - K_2)K_1x + K_2(K_1 - 2K_3)]x} \\ n_N &= n_O x \\ n_{O_2} &= \frac{(C_2 - C_1x)K_2K_3}{[(2K_3 - K_2)K_1x + K_2(K_1 - 2K_3)]x} \\ n_{N_2} &= K_1x^2n_{O_2}/K_2 \\ n_{NO} &= K_1xn_{O_2}/K_3 \end{aligned}\right\} \tag{9}$$

Then

$$\rho = K_3n_{NO}/n_On_N$$

$$p = \rho(n_O + n_N + n_{O_2} + n_{N_2} + n_{NO})R_OT$$

To determine the free parameter x, we substitute expressions (9) into the remaining equation, namely, Eq. (7), and obtain a cubic in x:

$$Ax^3 + Bx^2 + Cx + D = 0 \tag{10}$$

where

$$A = C_1K_3(2v_N - v_{N_2})/K_2$$

$$B = \frac{2C_1K_3v_O}{K_2} - h\left(\frac{2K_3}{K_2} - 1\right) + (C_1 - C_2)v_N - C_1v_{NO} + \frac{C_2K_3v_{NO}}{K_2}$$

$$C = h\left(\frac{2K_3}{K_1} - 1\right) - \frac{2C_2K_3v_N}{K_1} + (C_1 - C_2)v_O + C_2v_{NO} - \frac{C_1K_3v_{O_2}}{K_1}$$

$$D = C_2K_3(v_{O_2} - 2v_O)/K_1$$

The roots of the cubic (10) may be expressed in a closed form. Imaginary and negative roots are discarded. However, not every positive root leads to a physically meaningful solution. The mathematical constraints on x are the requirements that all five concentrations n_i be nonnegative. Equating the numerator of n_O in (9) to zero yields two values of x of which only one is positive. This positive value obviously corresponds to a mixture frozen at its low-temperature composition (minimum enthalpy). Similarly, the numerator

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* Consultant; also Associate Professor, Aeronautical, Astronautical, and Engineering Sciences, Purdue University, Lafayette, Ind. Member AIAA.

† Aerodynamics Engineer, Solid and Fluid Physics Department, Research and Development. Member AIAA.

of n_{O_2} equated to zero gives $x = C_2/C_1$. This value of x corresponds to a state of complete dissociation (maximum enthalpy). The zeros of the numerators of n_O and n_{O_2} are therefore the bounds of x. Calculations show the existence of not more than one root of Eq. (10) falling inside the determined bounds.

Inclusion of a neutral species, such as argon, would add one term in Eq. (7). Consequently, in the coefficients of the cubic, h would be replaced by $h - v_A n_A$. The algebra involved in carrying out the derivations becomes progressively more complicated as the number of reacting species is increased. The inclusion of ionization introduces higher-order algebraic nonlinearity. The authors have obtained explicit solutions in the case of coupled dissociation-ionization reactions (four species). Addition of molecular ionization, such as the $NO \rightleftharpoons NO^+ + e^-$ reaction, to the example discussed in this note leads to a sixth-degree polynomial in $y = (n_N/n_O)^{1/2}$, the roots of which have to be evaluated numerically. Ionization of atomic species in the presence of more than one dissociation reaction requires a simultaneous solution of two cubics. Thus the advantages of a closed-form solution are lost when the number of species is increased.

Although the existence of closed-form solutions is primarily of academic importance, the use of an exact solution, instead of an iteration scheme, results in considerable time savings in practical applications requiring a large number of computations.

Acceleration of Burning Rate of Composite Propellants by Sound Waves

ISIDOR ELIAS*
Acoustica Associates, Inc., Los Angeles, Calif.

AND

HENRY CHEUNG† AND NORMAN S. COHEN‡
Aerojet-General Corporation, Sacramento, Calif.

Nomenclature

ξ = particle displacement
p = acoustic pressure
ρ = gas density
c = gas-sound velocity
ω = angular sound frequency
δ = boundary-layer thickness
ν = gas-kinematic viscosity

Introduction

ONE serious limitation of solid-propellant rocket motors is that the thrust cannot be modulated upon command to meet in-flight contingencies. Thrust modulation by variation of nozzle-throat area, in a number of ways, has been under study for some time. An alternate approach to achieve thrust modulation is in-flight control of propellant burning rate. This paper is an account of an experimental study of using acoustics as a means for controlling the burning rate of composite solid propellants.

Theoretical Background

The concept of modulating the burning rate of composite solid propellants with sound waves is derived from the Summerfield[1] granular diffusion flame model.[1] It is hypothesized that pockets of gaseous fuel and oxidizer evolve from the hot propellant surface; the pockets then mix and react at some distance from the surface. This model suggests that controlling the rate of mixing of the gaseous pockets, under conditions where diffusion processes are rate-governing, would afford a means of controlling the burning rate. Plane travelling acoustic waves sweeping through the reaction zone would cause the gaseous pockets to follow the local oscillatory motion of the sound field so that more rapid mixing results. Accordingly, the burning rate should be accelerated, and modulation could be accomplished by variation of acoustic parameters.

There are three considerations that specify an appropriate acoustic field. One is that the sound field be of sufficient intensity so that the particle displacement can be large. In a plane travelling wave, the particle displacement is expressed by

$$\xi = p/\rho c\omega \tag{1}$$

Since the oxidizer and fuel pockets differ in (ρc), the displacements of each for a given sound field will differ also. This relative motion between the pockets will give rise to enhanced mixing.

A second factor to be considered is the viscous forces between the acoustic motion and the channel walls. This gives rise to a reduction in particle displacement within an acoustic boundary layer, the thickness of which is given by

$$\delta = (2\nu/\omega)^{1/2} \tag{2}$$

Efficient use of acoustic energy for mixing requires that the acoustic boundary layer be substantially less than the thickness of the reaction zone. This criterion imposes a lower limit on acoustic frequency according to Eq. (2). Finally, it is important that the gas pockets experience at least one complete oscillation while traversing the mixing zone. If one used estimated values for the pertinent combustion parameters it, was indicated in Refs. 1 and 2, that a sound field of 10 kc/sec at a sound pressure level of 174 db would be satisfactory.

Experimental Program

A schematic drawing of the experimental apparatus and instrumentation is shown in Fig. 1. All of the rocket-motor firings were made with internal-burning grains in chambers 5 in. in diameter by 20 in. in length, containing approximately 15 lb of an ammonium-perchlorate polyurethane-type composite propellant. Details of the instrumentation, firing procedure, and a sequential description of the test programs may be found in Refs. 2 and 3.

A Levavasseur whistle was used as the sound generator. Various gases under pressure could be fed through the whistle to provide the desired sound field. The output of the whistle was coupled to the rocket motor through an exponential horn to provide essentially plane wave propagation inside the motor cavity. Details of the whistle operation and performance are given in Ref. 4.

Experimental Results

Initially the sound field was produced by exciting the whistle with nitrogen gas at 9.6 and 14 kc/sec, each at 174 db. Thirty-nine firings were made with essentially no effect on the burning rate. It was decided that a gas having a

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* Research Physicist; now at North American Aviation, Inc., Rocketdyne Division, Canoga Park, Calif. Member AIAA.

† Senior Research Engineer; now at North American Aviation, Inc., Space and Information Systems Division, Downey, Calif. Member AIAA.

‡ Development Engineer, Propellant Ballistics Laboratory, Solid Rocket Plant. Associate Member AIAA.